

Higgs Boson Mass From Gauge-Higgs Unification

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Abstract

In certain five dimensional gauge theories the Standard Model Higgs doublet is identified, after compactification on the orbifold S^1/Z_2 , with the zero mode of the fifth component of the gauge field. An effective potential for the Higgs field is generated via quantum corrections, triggered by the breaking of the underlying gauge symmetry through boundary conditions. The quartic Higgs coupling can be estimated at low energies by employing the boundary condition that it vanishes at the compactification scale Λ , as required by five dimensional gauge invariance. For $\Lambda \gtrsim 10^{13} - 10^{14}$ GeV, the Standard Model Higgs boson mass is found to be $m_H = 125 \pm 4$ GeV, corresponding to a top quark pole mass $M_t = 170.9 \pm 1.8$ GeV. A more complete (gauge-Higgs-Yukawa) unification can be realized for $\Lambda \sim 10^8$ GeV, which happens to be the scale at which the SU(2) weak coupling and the top quark Yukawa coupling have the same value. For this case, $m_H = 117 \pm 4$ GeV.

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It seems reasonable to hope that the Standard Model (SM) Higgs boson will likely be found in the near future, most likely at the LHC. The discovery should reveal a great deal about the origin of electroweak breaking and the mechanism responsible for generating the quark and charged lepton masses. A precise measurement of the Higgs mass m_H should help distinguish between various competing theoretical ideas. One could argue, for example, that the MSSM would be one of the favored schemes if m_H turns out to be close to its current experimental lower limit of 114.4 GeV [1]. However, values of m_H around 125 GeV or larger, would suggest a much more serious consideration of other competing ideas. For instance, in a class of higher dimensional supersymmetric orbifold models in which the 4D N=1 supersymmetry is broken at M_{GUT} , the Higgs mass $m_H = 145(\pm 19)$ GeV [2]. The SM gauge couplings in these models are unified at M_{GUT} by employing a non-canonical $U(1)_Y$. An important extension of these ideas implements gauge and Yukawa coupling unification at M_{GUT} [3]. For instance, with gauge-top quark Yukawa coupling unification and with SUSY broken at M_{GUT} , the SM Higgs boson mass turns out to be 135 ± 6 GeV [3]. Somewhat larger values for the Higgs mass, 144 ± 4 GeV, are found with gauge-bottom quark Yukawa coupling unification [3].

In this letter we attempt to estimate m_H by employing the idea of gauge-Higgs unification (GHU) which has attracted a fair amount of recent attention [4]-[12]. We consider, in particular, 5D models compactified on an orbifold S^1/Z_2 , such that the zero mode of the fifth component of the bulk gauge field can be identified with the SM Higgs doublet. The so-called "gauge-Higgs" condition, to be explained shortly, enables us to estimate the SM Higgs mass m_H . Using two-loop renormalization group equations (RGEs), we find that m_H exceeds the LEP2 lower bound if the compactification scale $\Lambda \gtrsim 10^6$ GeV. The weak $SU(2)$ gauge coupling and the top Yukawa coupling have the same magnitude at scales close to 10^8 GeV. If the latter is identified with the compactification scale, the Higgs mass m_H is predicted to be 117 ± 4 GeV. Finally, following [2, 3], if Λ is identified with the SM gauge coupling unification scale of order 4×10^{16} GeV which is possible with non-canonical $U(1)_Y$, $m_H = 125 \pm 4$ GeV.

We consider 5D Gauge-Higgs Unification (GHU) model with the fifth dimension compactified on the orbifold S^1/Z_2 which yields a chiral "low energy" theory in four dimensions. In GHU models, the 5D bulk gauge symmetry is broken down to the SM by imposing suitable boundary conditions. The SM Higgs doublet emerges as a zero-mode of the fifth component of the higher dimensional gauge field. The higher dimensional gauge symmetry prevents the appearance of a tree level scalar potential. However, since the bulk gauge symmetry is broken by the boundary condition, a quartic Higgs potential is induced through quantum correction. In particular, at one-loop level, the effective Higgs potential has been found to be finite [6]. This finiteness can be interpreted as a remnant of the higher dimensional gauge invariance and its

non-local breaking by the boundary condition. The mass scale of the finite effective potential is controlled by the Kaluza-Klein (KK) mass, $1/R$, where R is the radius of the fifth dimension.

Recently, a new phenomenological treatment of GHU models has been proposed [8]. It has been shown that the effective SM Higgs quartic coupling λ calculated in a given GHU model coincides with the one radiatively generated in the effective low energy theory (without a quartic coupling at tree level), provide the compactification scale $\Lambda = 1/(2\pi R)$ is identified with the cutoff scale in evaluating quantum corrections. This implies that the higher dimensional gauge invariance is restored at scales smaller than the extra dimensional volume $2\pi R$, such that there is no effective quartic Higgs coupling at high energy, it appears, should be applicable to any GHU model. Thus, using renormalization group equations (RGEs), we can evaluate the SM quartic Higgs coupling by requiring that λ vanishes at $\Lambda = 1/(2\pi R)$. This boundary condition for λ is called [8] the “gauge-Higgs condition”. It is reminiscent of the so-called vacuum stability bound on m_H [13].

Corrections to the Higgs mass squared in GHU models, on the other hand, are very much dependent on the particle content and imposed boundary conditions (see, for instance [8]), and so in this paper we can treat the Higgs mass² as a free parameter in the low energy effective theory, to be suitably adjusted to yield the desired electroweak symmetry breaking.

In contrast, as we previously discussed, the gauge-Higgs condition is a model-independent condition imposed on the low energy effective theory. Therefore, when applied to the SM Higgs doublet, we can obtain a prediction for the physical Higgs boson mass as a function of the compactification scale. The Higgs boson mass prediction we will obtain is magnitude wise the same as the vacuum stability bound mentioned above. However, we have a physics interpretation for it, namely the GHU model(s) provides an ultraviolet completion of the SM. In the bottom-up picture, the quartic Higgs coupling in the SM reaches zero at the compactification scale, at which point the GHU model takes over. The Higgs potential is smoothed away such that no instability in the Higgs potential occurs.

We are now ready to discuss the physical Higgs boson mass m_H in this setup. With the electroweak symmetry breaking property implemented, m_H is determined by its quartic coupling. Imposing the “gauge-Higgs condition” for the Higgs quartic coupling at a given compactification scale $\Lambda (= 1/(2\pi R))$ and solving the two loop RGEs, [14], towards the electroweak scale, we obtain the Higgs boson mass as a function of Λ . Namely,

$$m_H(m_H) = \sqrt{\lambda(m_H)} v. \quad (1)$$

For the three SM gauge couplings, we have

$$\frac{dg_i}{d\ln\mu} = \frac{b_i}{16\pi^2} g_i^3 + \frac{g_i^3}{(16\pi^2)^2} \sum_{j=1}^3 B_{ij} g_j^2, \quad (2)$$

where μ is the renormalization scale, g_i ($i = 1, 2, 3$) are the SM gauge couplings and

$$b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right), \quad b_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{6}{2} & -26 \end{pmatrix}. \quad (3)$$

The top quark pole mass is taken to be $M_t = 170.9 \pm 1.8$ GeV, [15], with $(\alpha_1, \alpha_2, \alpha_3) = (0.01681, 0.03354, 0.1176)$ at M_Z [16]. For the top Yukawa coupling y_t , [14],

$$\frac{dy_t}{d \ln \mu} = y_t \left(\frac{1}{16\pi^2} \beta_t^{(1)} + \frac{1}{(16\pi^2)^2} \beta_t^{(2)} \right). \quad (4)$$

Here the one-loop contribution is

$$\beta_t^{(1)} = \frac{9}{2} y_t^2 - \left(\frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2 \right), \quad (5)$$

while the two-loop contribution is given by

$$\begin{aligned} \beta_t^{(2)} &= -12 y_t^4 + \left(\frac{393}{80} g_1^2 + \frac{225}{16} g_2^2 + 36 g_3^2 \right) y_t^2 \\ &+ \frac{1187}{600} g_1^4 - \frac{9}{20} g_1^2 g_2^2 + \frac{19}{15} g_1^2 g_3^2 - \frac{23}{4} g_2^4 + 9 g_2^2 g_3^2 - 108 g_3^4 \\ &+ \frac{3}{2} \lambda^2 - 6 \lambda y_t^2. \end{aligned} \quad (6)$$

In solving Eq.(4) from M_t to the compactification scale Λ , the initial top Yukawa coupling at $\mu = M_t$ is determined from the relation between the pole mass and the running Yukawa coupling [17],

$$\frac{M_t}{m_t(M_t)} \simeq 1 + \frac{4}{3} \frac{\alpha_3(M_t)}{\pi} + 10.91 \left(\frac{\alpha_3(M_t)}{\pi} \right)^2, \quad (7)$$

with $y_t(M_t) = \sqrt{2} m_t(M_t)/v$, where $v = 246.2$ GeV.

The RGE for the Higgs quartic coupling is given by [14],

$$\frac{d\lambda}{d \ln \mu} = \frac{1}{16\pi^2} \beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2} \beta_\lambda^{(2)}, \quad (8)$$

with

$$\beta_\lambda^{(1)} = 12\lambda^2 - \left(\frac{9}{5} g_1^2 + 9 g_2^2 \right) \lambda + \frac{9}{4} \left(\frac{3}{25} g_1^4 + \frac{2}{5} g_1^2 g_2^2 + g_2^4 \right) + 12 y_t^2 \lambda - 12 y_t^4, \quad (9)$$

and

$$\begin{aligned}
\beta_\lambda^{(2)} = & -78\lambda^3 + 18 \left(\frac{3}{5}g_1^2 + 3g_2^2 \right) \lambda^2 - \left(\frac{73}{8}g_2^4 - \frac{117}{20}g_1^2g_2^2 + \frac{2661}{100}g_1^4 \right) \lambda - 3\lambda y_t^4 \\
& + \frac{305}{8}g_2^6 - \frac{289}{40}g_1^2g_2^4 - \frac{1677}{200}g_1^4g_2^2 - \frac{3411}{1000}g_1^6 - 64g_3^2y_t^4 - \frac{16}{5}g_1^2y_t^4 - \frac{9}{2}g_2^4y_t^2 \\
& + 10\lambda \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \right) y_t^2 - \frac{3}{5}g_1^2 \left(\frac{57}{10}g_1^2 - 21g_2^2 \right) y_t^2 - 72\lambda^2 y_t^2 + 60y_t^6. \quad (10)
\end{aligned}$$

In Figure 1, we plot the Higgs boson mass m_H as a function of the compactification scale for an input top quark pole mass $M_t = 170.9 \pm 1.8$ GeV. The current Higgs boson mass bound, $m_H > 114.4$ GeV, from LEP2 [1] is displayed as horizontal line. Requiring the compactification scale $\Lambda \lesssim M_{Pl} = 1.2 \times 10^{19}$ GeV, the upper bound on the Higgs boson mass is found to be $m_H \lesssim 129$ GeV. A compactification scale larger than 2.5×10^6 GeV results in a Higgs boson mass which is consistent with the current lower bound. Higgs boson masses for varying top quark pole mass are

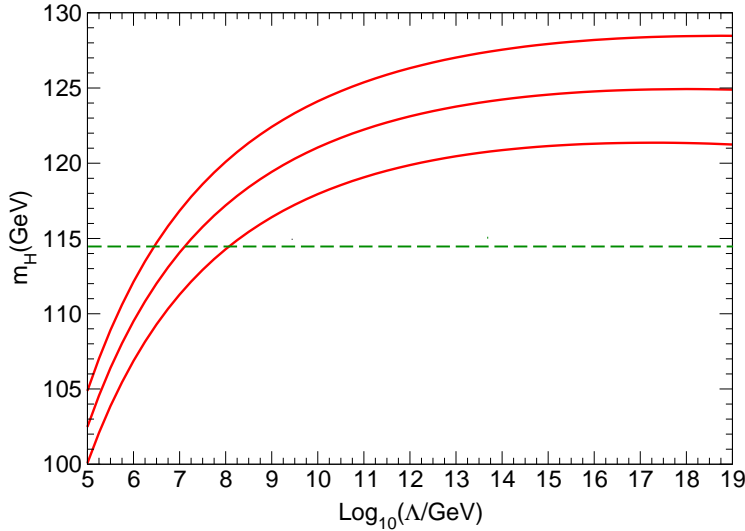


Figure 1: Higgs boson mass prediction versus the compactification scale, $\text{Log}_{10}(\Lambda/\text{GeV})$. The solid lines (in red) correspond from bottom to top to input top quark pole masses, $M_t = 169.1, 170.9$ and 172.8 GeV. The horizontal line shows the current Higgs mass bound, $m_H \geq 114.4$ GeV, from LEP2.

In the (simplest) GHU model, the unification of gauge and Yukawa interactions would imply fermion mass coincide with the weak gauge boson mass at Λ . With the possible exception of the top quark, as we will see, this is clearly not acceptable. To realize the hierarchy of fermion masses in the SM, more elaborate GHU model must be considered. There have been various efforts along this direction [11]. As far as the top quark is concerned the running top Yukawa coupling and the $\text{SU}(2)$ gauge coupling meet at an intermediate scale (see Figure 2), and it would

Λ [GeV]	$M_t = 169.1$ GeV	$M_t = 170.9$ GeV	$M_t = 172.8$ GeV
10^5	100.0	102.4	104.9
10^7	111.3	114.1	116.9
10^9	116.4	119.4	122.5
10^{11}	119.1	122.2	125.4
10^{13}	120.5	123.8	127.1
10^{15}	121.1	124.6	128.0
10^{17}	121.4	124.9	128.4
10^{19}	121.2	124.9	128.5

Table 1: Higgs boson masses (in GeV) for varying M_t and Λ .

be natural to identify this 'merger' point with the compactification scale. This observation allows us to realize gauge-Higgs and gauge-top Yukawa coupling unification at Λ , [12], in this case more precise prediction for the Higgs boson mass is obtained, namely $m_H = 117 \pm 4$ GeV (see Table 2).

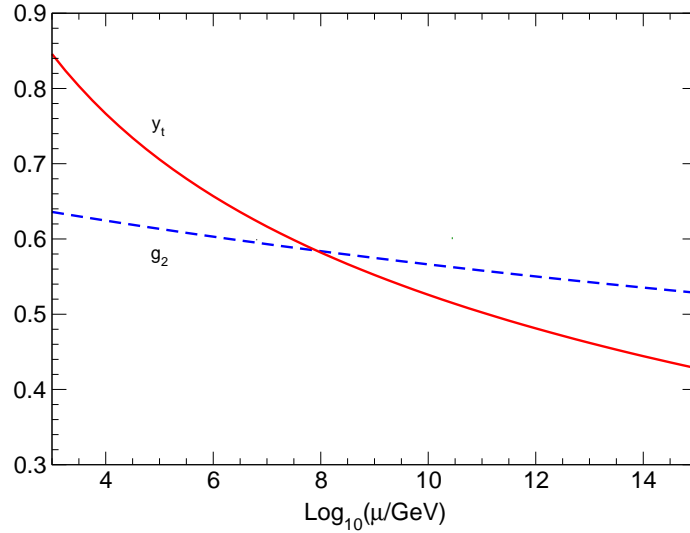


Figure 2: Plot of SU(2) gauge $g_2(\mu)$ (dashed line) and top Yukawa $y_t(\mu)$ (solid line) coupling versus $\text{Log}_{10}(\mu/\text{GeV})$. The two couplings merge at $\mu = 8.41 \times 10^7$ GeV, for $M_t = 170.9$ GeV.

Another interesting possibility is to identify Λ with $M_{\text{GUT}} \sim 10^{14} - 10^{16}$ GeV. In this case, $m_H = 125 \pm 4$ GeV. Although the three SM gauge couplings do not meet at M_{GUT} with a

	$M_t = 169.1$ GeV	$M_t = 170.9$ GeV	$M_t = 172.8$ GeV
Λ [GeV]	3.26×10^7	8.41×10^7	2.34×10^8
m_H [GeV]	112.9	117.0	121.1

Table 2: Higgs boson mass (in GeV) for varying M_t with $g_2(\Lambda) = y_t(\Lambda)$ at $\mu = \Lambda$ (see Figure 2).

canonical normalization of $5/3$ for $U(1)_Y$, a different choice, say $4/3$, can lead to gauge coupling unification at $M_{\text{GUT}} = 4 \times 10^{16}$ GeV [2, 3]. Note that the upper bound on m_H we have found for GHU models is well below the bound of 180 GeV or so, obtained from the entirely different requirement that λ should remain perturbative between M_Z and M_{Pl} .

In conclusion, we have considered gauge-Higgs unification in five dimensions with S^1/Z_2 orbifold compactification, such that the SM Higgs doublet emerges as the fifth component of the higher dimensional gauge field. Due to higher dimensional gauge invariance, there is no tree level Higgs potential in the effective four dimensional theory. This is modified by quantum corrections which generate a quartic Higgs potential with an effective finite cutoff, $\Lambda = 1/(2\pi R)$. While the induced Higgs self energy is highly dependent on the details of the model considered, the quartic Higgs coupling coincides with the one obtained in the low energy massless theory by employing the gauge-Higgs condition at Λ . Imposing this condition on the Higgs quartic coupling in the SM and employing two loop RGEs, we have obtained predictions for the Higgs boson mass as a function of the compactification scale.

Finally, a comment on cosmology is in order here. The existence of (non-baryonic) dark matter has been established from various observations of the present universe. Except for the Higgs sector, the gauge-Higgs unification model shares the same structure as the Universal Extra Dimension model [19, 18], so that the first KK excitation of $U(1)_Y$ gauge boson is a plausible dark matter candidate [20]. In our case, the mass of this particle can be very high and, indeed, it can exceed the unitarity limit on the mass of cold dark matter as a thermal relic. However, there are various possibilities to realize superheavy dark matter, such as its production through inflaton decay.

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References

- [1] R. Barate *et al.* [LEP Working Group for Higgs boson searches], Phys. Lett. B **565**, 61 (2003).

- [2] V. Barger, J. Jiang, P. Langacker and T. Li, Phys. Lett. B **624**, 233 (2005); Nucl. Phys. B **726**, 149 (2005); Model with non-canonical normalization for $U(1)_Y$ have also been discussed by I. Gogoladze, T. Li, V. N. Senoguz and Q. Shafi, Phys. Rev. D **74**, 126006 (2006); I. Gogoladze, C. A. Lee, T. Li and Q. Shafi, arXiv:0704.3568 [hep-ph]; A. Kehagias and N.D. Tracas, arXiv:hep-ph/0506144; A. Aranda, J. L. Diaz-Cruz and A. Rosado, Int. J. Mod. Phys. A **22**, 1417 (2007).
- [3] I. Gogoladze, T. Li and Q. Shafi, Phys. Rev. D **73**, 066008 (2006)
- [4] N. S. Manton, Nucl. Phys. B **158**, 141 (1979); D. B. Fairlie, J. Phys. G **5**, L55 (1979); Phys. Lett. B **82**, 97 (1979).
- [5] Y. Hosotani, Phys. Lett. B **126**, 309 (1983); Phys. Lett. B **129**, 193 (1983); Phys. Rev. D **29**, 731 (1984); Annals Phys. **190**, 233 (1989).
- [6] N. V. Krasnikov, Phys. Lett. B **273**, (1991), 246; H. Hatanaka, T. Inami and C. S. Lim, Mod. Phys. Lett. A **13**, (1998), 2601; G. R. Dvali, S. Randjbar-Daemi and R. Tabbash, Phys. Rev. D **65**, (2002), 064021; N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **513**, (2001), 232; I. Antoniadis, K. Benakli and M. Quiros, New J. Phys. **3**, (2001), 20.
- [7] M. Kubo, C. S. Lim and H. Yamashita, Mod. Phys. Lett. A **17** (2002), 2249; C. A. Scrucca, M. Serone and L. Silvestrini, Nucl. Phys. B **669** (2003), 128; C. A. Scrucca, M. Serone, L. Silvestrini and A. Wulzer, JHEP **0402**, 049 (2004); G. Panico and M. Serone, JHEP **0505**, 024 (2005) N. Haba, Y. Hosotani, Y. Kawamura and T. Yamashita, Phys. Rev. D **70** (2004), 015010; N. Haba and T. Yamashita, JHEP **0404** (2004) 016; Y. Hosotani, S. Noda and K. Takenaga, Phys. Rev. D **69** (2004), 125014; Phys. Lett. B **607** (2005), 276; K. W. Choi, N. Y. Haba, K. S. Jeong, K. I. Okumura, Y. Shimizu and M. Yamaguchi, JHEP **0402**, 037 (2004); A. Demaria and K. L. McDonald, Phys. Rev. D **75**, 056006 (2007).
- [8] N. Haba, S. Matsumoto, N. Okada and T. Yamashita, JHEP **0602**, 073 (2006).
- [9] M. Sakamoto and K. Takenaga, Phys. Rev. D **75**, 045015 (2007).
- [10] N. Maru and T. Yamashita, Nucl. Phys. B **754**, 127 (2006).
- [11] For example, see G. Cacciapaglia, C. Csaki and S. C. Park, JHEP **0603**, 099 (2006), and references therein.

- [12] I. Gogoladze, Y. Mimura and S. Nandi, Phys. Lett. B **560**, 204 (2003); Phys. Lett. B **562**, 307 (2003); Phys. Rev. Lett. **91**, 141801 (2003); Phys. Rev. D **69**, 075006 (2004); I. Gogoladze, T. Li, Y. Mimura and S. Nandi, Phys. Rev. D **72**, 055006 (2005); I. Gogoladze, C. A. Lee, Y. Mimura and Q. Shafi, arXiv:hep-ph/0703107.
- [13] N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B **158**, 295 (1979); P. Q. Hung, Phys. Rev. Lett. **42**, 873 (1979); M.A.B. Beg, C. Panagiotakopoulos and A. Sirlin, Phys. Rev. Lett. **52** (1984) 883; M. Lindner, Z. Phys. C **31**, 295 (1986); M. Sher, Phys. Rept. **179**, 273 (1989); G. Altarelli and G. Isidori, Phys. Lett. B **337**, 141 (1994); J. A. Casas, J. R. Espinosa and M. Quiros, Phys. Lett. B **342**, 171 (1995); Phys. Lett. B **382**, 374 (1996); J. R. Espinosa and M. Quiros, Phys. Lett. B **353**, 257 (1995).
- [14] M. E. Machacek and M. T. Vaughn, Nucl. Phys. B **222**, 83 (1983); Nucl. Phys. B **236**, 221 (1984); Nucl. Phys. B **249**, 70 (1985); C. Ford, I. Jack and D. R. T. Jones, Nucl. Phys. B **387** (1992) 373, [Erratum-ibid. **B504** (1997) 551]; V. D. Barger, M. S. Berger and P. Ohmann, Phys. Rev. D **47**, 1093 (1993); M. X. Luo and Y. Xiao, Phys. Rev. Lett. **90**, 011601 (2003).
- [15] [CDF Collaboration], arXiv:hep-ex/0703034.
- [16] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33** (2006) 1.
- [17] See, for example, H. Arason, D. J. Castano, B. Keszthelyi, S. Mikaelian, E. J. Piard, P. Ramond and B. D. Wright, Phys. Rev. D **46**, 3945 (1992).
- [18] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
- [19] T. Appelquist, H. C. Cheng and B. A. Dobrescu, Phys. Rev. D **64**, 035002 (2001).
- [20] H. C. Cheng, K. T. Matchev and M. Schmaltz, Phys. Rev. D **66**, 036005 (2002).